

# Phenomenological features in a model with non-universal gaugino $CP$ phases

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**Abstract.** We study phenomenological features in an extended gauge mediation SUSY breaking model that has non-universal gaugino masses and  $CP$  phases. We show that large  $CP$  phases in soft SUSY breaking parameters can be consistent with the constraints coming from the electric dipole moment (EDM) of an electron, a neutron, and also a mercury atom. Masses of the superpartners are not necessarily required to be larger than 1 TeV but allowed to be  $O(100)$  GeV. We also investigate the mass spectrum of Higgs scalars and their couplings to gauge bosons in that case. Compatibility of this model with the present experimental data on the Higgs sector is discussed.

## 1 Introduction

In supersymmetric extensions of the standard model, new  $CP$  phases are generally introduced through supersymmetry (SUSY) breaking. Although these  $CP$  phases could play an interesting phenomenological role related to cosmological baryon number asymmetry, for example, it is well known that the electric dipole moments (EDMs) of an electron and a neutron [1–3] impose severe constraints on such  $CP$  phases of soft SUSY breaking parameters in the minimal supersymmetric standard model (MSSM) [4–8]. It seems to be very important to examine these constraints because of their phenomenological consequences.

Some possibilities to overcome these constraints have been proposed by now. In the first type solution, the soft SUSY breaking parameters are taken to be  $O(100)$  GeV by assuming that the soft  $CP$  phases are smaller than  $10^{-2}$  [4–7]. Since such small phases are not protected by any symmetry, it is usually considered to be unnatural, and regarded as a  $CP$  problem in the MSSM. In the second one, the soft  $CP$  phases are supposed to be  $O(1)$ , while a part of the relevant soft SUSY breaking parameters are assumed to be  $O(1)$  TeV or larger<sup>1</sup>. However, considering SUSY breaking to be larger than  $O(1)$  TeV seems to

be unattractive from a viewpoint of weak scale SUSY. It may also be difficult to expect any phenomenological effects through the present and near future experiments in this case.

As the third possibility, we may expect the cancellation among various contributions to the EDMs [11–16]. If such a cancellation occurs and both the  $CP$  phases of  $O(1)$  and the soft SUSY breaking parameters of  $O(100)$  GeV can be consistent with the EDM constraints, we might have a lot of interesting phenomenology at the weak scale [15–26]. If we consider the origin of the baryon number asymmetry in the universe to be due to electroweak baryogenesis, for example, it will be necessary to introduce some new sources of  $CP$  violation. It is known that the Cabibbo–Kobayashi–Maskawa (CKM) phase in the standard model (SM) is insufficient to explain the baryon number asymmetry because of a suppression due to the smallness of the quark flavor mixing [27]. If there exist large  $CP$  phases in the soft SUSY breaking parameters, the requirement for the electroweak phase transition to be strongly first order might be relaxed and the required Higgs mass bound could be larger [28–34]. Various SUSY leptogenesis scenarios also seem to require large  $CP$  phases in the soft SUSY breaking parameters [35–38]. Thus, the existence of such  $CP$  phases is a fascinating possibility from the viewpoint that they present us with promising sources for the  $CP$  violation required in baryogenesis and leptogenesis. Moreover, such  $CP$  phases might be checked through LHC experiments.

Various works on this third possibility have suggested that the constraints on the EDMs of an electron and a neutron could be satisfied even in the case that the  $CP$  phases in the soft SUSY breakings are  $O(1)$  and the superpartners are rather light. It is based on the effective cancella-

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<sup>1</sup> Various possibilities have been suggested. One of these is to assume that the sfermions in the first and second generation have heavy masses of  $O(1)$  TeV [8]. In another one, the  $A$  parameters are assumed to be non-universal and those related to the first and second generation are supposed to be very small, such as  $A_f = (0, 0, A)$  [9, 10]. In this case, one needs to assume  $\arg(\mu) < 10^{-2}$  and then the smallness of the  $CP$  phase is partially required as in the first solution [9, 10].

tion among various contributions to the EDMs<sup>2</sup>. On the other hand, there is another claim: if we add the constraint from the EDM of the mercury atom, the allowed parameter regions disappear. It suggests that the parameter region for their cancellation are different for electron and mercury [39]. However, it is useful to note that the usual analyses of the EDMs are based on the assumption of universal gaugino masses as stressed in [15, 16]. If we do not make this assumption, we may find the way out of this difficulty.

Since the gaugino masses are universal in the usual SUSY breaking scenario, it may be considered that such an assumption is unrealistic. However, non-universal gaugino masses can be realized naturally, if we consider, for example, the intersecting D-brane model [15, 16, 42], extended gauge mediation SUSY breaking [43, 44], and the SUSY breaking mediated by Abelian gaugino kinetic term mixing [45, 46]. In the previous paper [47], we examined the possibility of the reconciliation between the  $CP$  phases of  $O(1)$  and the experimental EDM constraints in a model with non-universal gaugino masses. In that study we showed that the EDM constraints could be satisfied in rather large regions of the SUSY breaking parameter space where large  $CP$  phases would exist, as long as there are physical  $CP$  phases in the gaugino masses. However, since the allowed parameter regions tend to be obtained for small  $\tan\beta$  [47], Higgs phenomenology might constrain the model strongly through the present Higgs search [19–25, 48].

In this paper we extend the study to the Higgs sector using the parameter regions allowed by the constraints from the EDMs of the electron, the neutron, and the mercury atom. We discuss the consistency of the scenario with the Higgs phenomenology. The paper is organized as follows. In Sect. 2 we introduce the model for the soft SUSY breaking with non-universal gaugino masses. In Sect. 3 we briefly describe the EDM of the mercury atom as an example of the EDM calculation. A numerical analysis of the EDM constraints is carried out by using the renormalization group. We apply this result to the estimation of the masses of the neutral Higgs scalar and the couplings between the Higgs scalars and the gauge bosons. We also discuss the predicted values of  $g-2$  of the muon and the electron. Section 4 is devoted to a summary.

## 2 A model with non-universal gaugino $CP$ phases

We briefly introduce the model with non-universal gaugino masses studied in this paper and fix the notation. We

<sup>2</sup> In the case of the EDM of the electron, the cancellation between the chargino contribution and the neutralino contribution has been shown to occur [11–13, 15, 16]. On the other hand, for the EDM of the neutron (EDMN) it is known that there are several types of cancellation; that is, there is cancellation between the diagrams of the gluino exchange and the chargino exchange diagrams and also cancellation among the gluino exchange diagrams themselves etc. [11, 12, 14]. In the case of the EDMN, the combined effect of these cancellations allows for large soft  $CP$  phases [11–13, 15, 16].

consider an extension of the well known minimal gauge mediation SUSY breaking (GMSB) scenario, which is defined by the following superpotential for the messenger fields [43, 44]:

$$W_m = \lambda_q \hat{S}_1 \hat{q} \hat{q} + \lambda_\ell \hat{S}_2 \hat{\ell} \hat{\ell}, \quad (1)$$

where  $\hat{q}$  and  $\hat{q}$  are  $\mathbf{3}$  and  $\mathbf{3}^*$  of  $SU(3)_c$  and  $\hat{\ell}$  and  $\hat{\ell}$  are the doublets of  $SU(2)_L$ . If both the singlet fields  $\hat{S}_1$  and  $\hat{S}_2$  couple with the hidden sector where SUSY breaks down,  $\hat{q}, \hat{q}$  and  $\hat{\ell}, \hat{\ell}$  play the role of messenger fields as in the case of the ordinary scenario [49–51]; for a review, see [52]. The only difference from the ordinary minimal GMSB scenario is that  $\hat{q}, \hat{q}$  and  $\hat{\ell}, \hat{\ell}$  couple with the different singlet chiral superfields  $\hat{S}_1$  and  $\hat{S}_2$  in the superpotential  $W_m$ . It is realized if we impose a suitable discrete symmetry on the model [43, 44]. If both their scalar components  $S_\alpha$  and their auxiliary components  $F_{S_\alpha}$  obtain vacuum expectation values (VEVs) due to the couplings with the SUSY breaking sector, the masses of the gauginos and the scalars in the MSSM are generated at one-loop and two-loop level, respectively. They are represented as functions of  $\Lambda_\alpha = \langle F_{S_\alpha} \rangle / \langle S_\alpha \rangle$  in a similar way as the ordinary scenario. However, the mass formulas are somewhat modified from the usual ones, since the messenger fields  $(\hat{q}, \hat{q})$  and  $(\hat{\ell}, \hat{\ell})$  couple with different singlets.

In this kind of model, the gaugino masses can be written in the form [43, 44]

$$M_3 = \frac{\alpha_3}{4\pi} \Lambda_1, \quad M_2 = \frac{\alpha_2}{4\pi} \Lambda_2, \quad M_1 = \frac{\alpha_1}{4\pi} \left( \frac{2}{3} \Lambda_1 + \Lambda_2 \right), \quad (2)$$

where  $\alpha_r = g_r^2/4\pi$  and  $g_r$  stands for the coupling constant for the standard model gauge group. These formulas show that  $M_3$  may be smaller than  $M_{1,2}$  in the case of  $\Lambda_2 > \Lambda_1$ . Since  $\Lambda_\alpha$  is generally independent, the phases contained in the gaugino masses are non-universal even in the case of  $|\Lambda_1| = |\Lambda_2|$ . In that case, we cannot remove them completely by using the  $R$ -transformation, unlike the case of universal gaugino masses. In fact, if we define the phases by  $\Lambda_\alpha \equiv |\Lambda_\alpha| e^{i\theta_\alpha}$  and make  $M_2$  real by the  $R$ -transformation, the phases of the gaugino masses  $M_r$  can be written as [43, 44]

$$\begin{aligned} \phi_3 &\equiv \arg(M_3) = \theta_1 - \theta_2, & \phi_2 &\equiv \arg(M_2) = 0, \\ \phi_1 &\equiv \arg(M_1) = \arctan \left( \frac{2|\Lambda_1| \sin(\theta_1 - \theta_2)}{3|\Lambda_2| + 2|\Lambda_1| \cos(\theta_1 - \theta_2)} \right). \end{aligned} \quad (3)$$

These formulas show that the phases of the gaugino masses can be parameterized by three parameters; that is,  $|\Lambda_1|$ ,  $|\Lambda_2|$  and  $\theta_1 - \theta_2$ .

The scalar masses are induced through the two-loop diagrams as in the ordinary case. Their formulas can be given

as [43, 44]

$$\begin{aligned} \tilde{m}_f^2 = & 2|A_1|^2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + \frac{2}{3} \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right] \\ & + 2|A_2|^2 \left[ C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right], \quad (4) \end{aligned}$$

where  $C_3 = 4/3$  and 0 for the SU(3) triplet and singlet fields, and  $C_2 = 3/4$  and 0 for the SU(2) doublet and singlet fields, respectively. The hypercharge  $Y$  is expressed as  $Y = 2(Q - T_3)$  by using both the electric charge  $Q$  and the diagonal SU(2) generator  $T_3$ . As is clear from this formula for the masses of the scalar superpartners, we have no FCNC problem induced by these soft scalar masses as in the ordinary case. This is the case even if we take account of the renormalization group effects since the running due to the renormalization groups occurs only for a narrow range.

We apply this soft SUSY breaking scenario to the MSSM framework. The MSSM superpotential contains the terms

$$\begin{aligned} W = & \sum_j \left( h_j^U \hat{H}_2 \hat{Q}_j \hat{U}_j + h_j^D \hat{Q}_j \hat{H}_1 \hat{D}_j + h_j^E \hat{L}_j \hat{H}_1 \hat{E}_j \right) \\ & + \mu \hat{H}_1 \hat{H}_2, \quad (5) \end{aligned}$$

where we take the Yukawa coupling diagonal basis for the quarks and the leptons. All Yukawa couplings  $h_j^f$  are supposed to be real. The Higgsino mass parameter  $\mu$  is generally complex. The soft SUSY breaking terms corresponding to the superpotential (5) are introduced by<sup>3</sup>

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & \sum_{\alpha} \tilde{m}_{\alpha}^2 |\phi_{\alpha}|^2 - \left[ \sum_j \left( A_j^U h_j^U H_2 \tilde{Q}_j \tilde{U}_j \right. \right. \\ & \left. \left. + A_j^D h_j^D \tilde{Q}_j H_1 \tilde{D}_j + A_j^E h_j^E \tilde{L}_j H_1 \tilde{E}_j \right) \right. \\ & \left. - B\mu H_1 H_2 - \frac{1}{2} \sum_r M_r \lambda_r \lambda_r + \text{h.c.} \right], \quad (6) \end{aligned}$$

where we put a tilde for the superpartners of the chiral superfields corresponding to the standard model contents. The first term represents the soft SUSY breaking masses for all scalar components of the MSSM chiral superfields. They are assumed to be given by (4). The third term in the brackets represents the gaugino mass terms, which are supposed to be given by (2). The soft SUSY breaking parameters  $B$  and  $A_j^f$  are the coefficients of the bilinear and trilinear scalar couplings with dimension of mass.

In the minimal GMSB model, as discussed in [53], the soft SUSY breaking parameters  $A_f$  and  $B$  can be induced through a radiative correction. In the case that  $A_f(\Lambda) = B(\Lambda) = 0$  is satisfied at the SUSY breaking scale  $\Lambda$ , which is expected in many GMSB scenarios,  $A_f$  and  $B$  are proportional to  $M_2$  at the low energy regions as a result of the

renormalization group effect. Thus, all of the  $CP$  phases in the soft SUSY breaking parameters are rotated away as long as the gaugino masses are universal [51, 53]. However, in the present case this situation is broken and there remain  $CP$  phases in the gaugino masses even in the case of  $A_f(\Lambda) = B(\Lambda) = 0$ , since the phases in the gaugino masses are not universal. The generation of the bare  $A_f$  and  $B$  is completely model dependent in this model as in the ordinary GMSB scenario. In the following study, we do not fix their origin and treat them as free parameters.

Here we make the additional assumption for the trilinear scalar couplings that they are proportional to the Yukawa couplings so as to satisfy the FCNC constraints. Although the soft SUSY breaking parameters  $A_j^f$ ,  $B$  and  $M_r$  may generally include  $CP$  phases, all of these are not independent physical phases. If we use the  $R$ -symmetry and redefine the fields appropriately, we can select the physical  $CP$  phases among them. We take them as

$$A_j = |A_j| e^{i\phi_{A_j}}, \quad \mu = |\mu| e^{i\phi_{\mu}}, \quad M_r = |M_r| e^{i\phi_r} \quad (r = 1, 3), \quad (7)$$

where  $B\mu$  and  $M_2$  are assumed to be real. Although the VEVs of the doublet Higgs scalars  $H_1$  and  $H_2$  are taken to be real in this definition at the tree level, a radiative correction could generally introduce  $CP$  phases to them. Taking account of this aspect and following [21], we define the VEVs of the doublet Higgs scalars  $H_1$  and  $H_2$  as

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}. \quad (8)$$

Finally, we summarize the model parameters related to the SUSY breaking. In the present framework, the free parameters related to the masses of the gauginos and the scalar superpartners are confined to  $A_1$  and  $A_2$ . Their phases are related to the physical phases  $\phi_3$  and  $\phi_1$  in (3). Since we assume universality for the  $A$  parameter such as  $A_j^f(\Lambda) = A$  at the SUSY breaking scale  $\Lambda$ , there remain five independent real parameters  $\phi_{\mu}$ ,  $\phi_A$ ,  $|\mu|$ ,  $|B|$ , and  $|A|$  in the sector of  $A_j^f$  and  $B$ . Thus, the model parameters relevant to the soft SUSY breaking are composed of eight real parameters,

$$|A_1|, \quad |A_2|, \quad |A|, \quad |B|, \quad |\mu|, \quad \phi_3, \quad \phi_A, \quad \phi_{\mu}. \quad (9)$$

The phase  $\xi$  in (8) will be determined by these parameters through minimizing the  $CP$ -violating Higgs potential [21].

### 3 Phenomenological effects of gaugino $CP$ phases

#### 3.1 Constraints from EDMs

In order to explain the constraints on the SUSY breaking parameters from the EDM, we at first take the case of mercury as an example to give a brief discussion. A detailed discussion of the EDMs of the electron and the neutron in the present model can be found in [47].

<sup>3</sup> We adopt the sign convention for  $\mu$ ,  $B$  and  $A_f$  to make the mass eigenvalues of quarks and leptons positive by a suitable field redefinition.



On the chargino and neutralino contributions  $\tilde{d}_f^X$  to the quark color EDM we remark that we can calculate it in the same way as the ordinary EDM of the electron [47]. We find that it can be written as

$$\begin{aligned} \tilde{d}_u^X &= \frac{\alpha m_u}{8\pi s_W^2} \left[ \frac{1}{m_u} \sum_{j,a} \frac{-2}{3m_j} G(x_{aj}) \operatorname{Im} \left( \mathcal{A}_{\chi_j^0}^{u_a} \right) \right. \\ &\quad \left. + \frac{1}{m_W \sin \beta} \sum_{j,a} \frac{1}{3m_j} G(x_{aj}) \operatorname{Im} \left( \mathcal{A}_{\chi_j^\pm}^{u_a} \right) \right], \\ \tilde{d}_d^X &= \frac{\alpha m_d}{8\pi s_W^2} \left[ \frac{1}{m_d} \sum_{j,a} \frac{1}{3m_j} G(x_{aj}) \operatorname{Im} \left( \mathcal{A}_{\chi_j^0}^{d_a} \right) \right. \\ &\quad \left. - \frac{1}{m_W \cos \beta} \sum_{j,a} \frac{2}{3m_j} G(x_{aj}) \operatorname{Im} \left( \mathcal{A}_{\chi_j^\pm}^{d_a} \right) \right]. \quad (22) \end{aligned}$$

In these formulas the mixing factors  $\mathcal{A}_{\chi^\pm}^f$  and  $\mathcal{A}_{\chi^0}^f$  are defined by

$$\begin{aligned} \mathcal{A}_{\chi_j^\pm}^{u_a} &= \frac{1}{\sqrt{2}} W_{1j}^{(+)} W_{2j}^{(-)} |V_{1a}^d|^2 \\ &\quad + \frac{1}{2 \cos \beta} \frac{m_d}{m_W} W_{1j}^{(+)} W_{1j}^{(-)} V_{2a}^{d*} V_{1a}^d, \\ \mathcal{A}_{\chi_j^\pm}^{d_a} &= \frac{1}{\sqrt{2}} W_{1j}^{(-)} W_{2j}^{(+)} |V_{1a}^u|^2 \\ &\quad + \frac{1}{2 \sin \beta} \frac{m_u}{m_W} W_{1j}^{(+)} W_{1j}^{(-)} V_{2a}^{u*} V_{1a}^u, \\ \mathcal{A}_{\chi_j^0}^{u_a} &= - \left[ \left( \frac{2}{9} t_W^2 U_{1j}^2 + \frac{2}{3} t_W U_{1j} U_{2j} \right) V_{1a}^{u*} V_{2a}^u \right. \\ &\quad \left. - \frac{m_u}{2m_W \sin \beta} \left\{ \left( \frac{1}{3} t_W U_{1j} U_{4j} + U_{2j} U_{4j} \right) |V_{1a}^u|^2 \right. \right. \\ &\quad \left. \left. - \frac{2}{3} t_W U_{1j} U_{4j} |V_{2a}^u|^2 \right\} \right], \\ \mathcal{A}_{\chi_j^0}^{d_a} &= - \left[ - \left( \frac{1}{9} t_W^2 U_{1j}^2 + \frac{1}{3} t_W U_{1j} U_{2j} \right) V_{1a}^{d*} V_{2a}^d \right. \\ &\quad \left. - \frac{m_d}{2m_W \cos \beta} \left\{ \left( \frac{1}{3} t_W U_{1j} U_{3j} - U_{2j} U_{3j} \right) |V_{1a}^d|^2 \right. \right. \\ &\quad \left. \left. + \frac{1}{3} t_W U_{1j} U_{3j} |V_{2a}^d|^2 \right\} \right], \quad (23) \end{aligned}$$

where  $t_W = \sin \theta_W / \cos \theta_W$ . We neglect the higher order terms of the quark mass in the expression of  $\mathcal{A}_{\chi^0}^f$ . Since the fermions in the external lines are very light compared with the fields in the internal lines,  $F(x)$  and  $G(x)$  are approximately written as

$$\begin{aligned} F(x) &= \frac{1-3x}{(1-x)^2} - \frac{2x^2}{(1-x)^3} \ln x, \\ G(x) &= \frac{1+x}{(1-x)^2} + \frac{2x}{(1-x)^3} \ln x. \quad (24) \end{aligned}$$

The gluino contribution is expected to be larger than the other contributions because of the strong coupling constant. If we expect cancellation among these contributions,

$\tilde{d}_f^g$  should be suppressed to have a magnitude similar to the others. In order to find the condition for it, we may estimate a factor  $\operatorname{Im}(\mathcal{A}_g^{f_a})$  in the case of  $|A| \gg |\mu|$ , for example. In that case it can be found to be approximately

$$\operatorname{Im}(\mathcal{A}_g^{f_a}) = O\left(\frac{m_{f_a} |A|}{M_2^2} \sin(\phi_3 - \phi_A)\right). \quad (25)$$

This shows that the existence of the gluino phase  $\phi_3$  may make it possible to suppress the gluino contribution to the level of the others. If this happens, the experimental bounds can be satisfied.

Both contributions of the charginos and the neutralinos are crucially affected by the relative magnitude of  $\mu$  and  $M_{1,2}$ . If  $|\mu| < |M_{1,2}|$  is satisfied, the Higgsino components dominate both the lightest neutralino and the lightest chargino. Although they are expected to yield the largest contribution to the EDM, Higgsino exchange effects can be suppressed due to the smallness of the Yukawa couplings. On the other hand, in the case of  $|M_1| < |\mu| < |M_2|$ , the lightest neutralino and the lightest chargino seem to be dominated by the bino and the Higgsinos, respectively. Since the gauge coupling  $g_1$  is larger than the relevant Yukawa couplings that determine the magnitude of their contribution, the chargino contribution can be suppressed in comparison with the neutralino contribution. As a result, they can yield contributions of similar order. If the latter situation for  $\mu$  and  $M_{1,2}$  is realized, the EDM constraint may be satisfied even in the case that the large  $CP$  phases exist in the soft SUSY breaking parameters. In the next part, we mainly focus our attention on such situations and carry out the numerical calculation.

### 3.2 Numerical results of the EDM constraints

At first we explain the procedure for the calculation. We evolve the soft SUSY breaking parameters from a certain SUSY breaking scale  $\Lambda$  to the weak scale by using the one-loop renormalization group equations (RGEs). There is an ambiguity on the scale where the soft SUSY breaking parameters are introduced and start their running. In the present analysis, we adopt  $\Lambda = \min(|A_1|, |A_2|)$  as such a scale, for simplicity. Since we mainly study the region where  $|A_2|/|A_1|$  is not so large, this prescription is not considered to affect the results largely. For the gauge and Yukawa coupling constants we use the two-loop RGEs. The RGEs from the unification scale  $M_U$  to  $\Lambda$  are composed of the SUSY ones for both the gauge and Yukawa coupling constants. The  $\beta$ -functions are calculated for the MSSM contents and the messenger fields. We solve these RGEs for various initial values of the Yukawa couplings at  $M_U$  and examine whether the masses of the top and bottom quarks and also the tau lepton are obtained at the weak scale. The messenger fields are supposed to decouple and the soft SUSY breaking parameters are introduced at  $\Lambda$ . Thus, the RGEs become the same as those of the MSSM below this scale.

In order to determine the phenomenologically interesting parameter regions, we impose several conditions on the

parameters at the weak scale obtained by the RGEs. As such conditions, we adopt the following items additionally to the above mentioned ones.

- (1) Various experimental mass bounds for the superpartners, such as gluinos, charginos, stops, staus, and charged Higgs scalars, should be satisfied. The color and the electromagnetic charge also should not be broken.
- (2) The physical true vacuum should be radiatively realized as the minimum of the scalar potential and satisfy  $\sin 2\beta = 2B\mu / (m_1^2 + m_2^2 + 2|\mu|^2)$ . As another true vacuum condition, moreover, we impose the consistency between this  $\sin 2\beta$  and the value of  $\tan\beta$  predicted from the Yukawa coupling and the top quark mass<sup>5</sup>. Only if the difference between them is sufficiently small, the parameters are accepted.

After restricting the parameter space at the high energy scale by imposing these conditions on the weak scale values, we finally calculate the EDMs of the electron, the neutron and the mercury atom. We compare these results with the present experimental bounds [54, 55],

$$|d_e/e| < 1.6 \times 10^{-27} \text{ cm}, \quad |d_n/e| < 0.3 \times 10^{-25} \text{ cm} \quad (26)$$

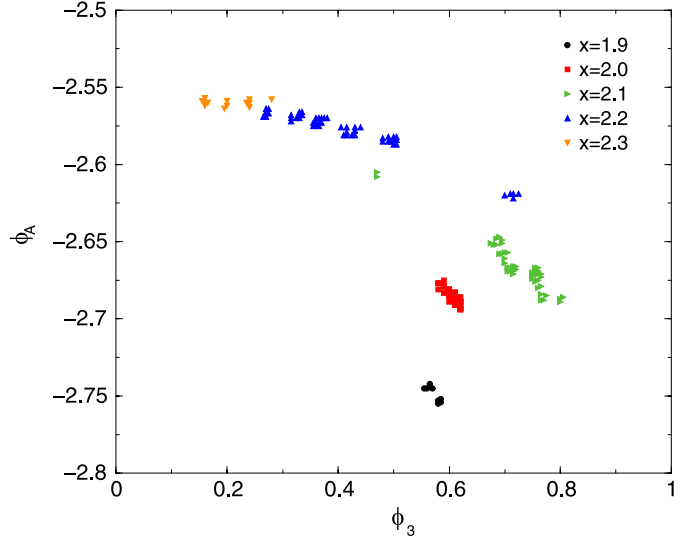
for the electron and the neutron and also (13) for the mercury.

We present the results of the numerical analysis, in which we fix some parameters to typical values such as  $|A_1| = 50 \text{ TeV}$ ,  $|\mu| = 100 \text{ GeV}$ , and  $\phi_\mu = -1.65$ , for simplicity. It seems hard to have consistent solutions for  $|A_1| \leq 35 \text{ TeV}$  and  $|A_1| \geq 55 \text{ TeV}$ . We adopt the value of  $\phi_\mu$  to introduce a seed for the large  $CP$  violation in the model. Since the one-loop RGEs do not make the phase run largely, this input value is equal to the weak scale one. We also tune the initial value of  $|B|$  so as to realize  $\tan\beta = 3.85$ , since only very restricted values of  $\tan\beta$  like 3.5–4 seem to be consistent with the EDM constraints. Under these settings, we search the parameter regions that satisfy the above mentioned phenomenological conditions by scanning the remaining parameters through the following ranges at the scale  $A$ :

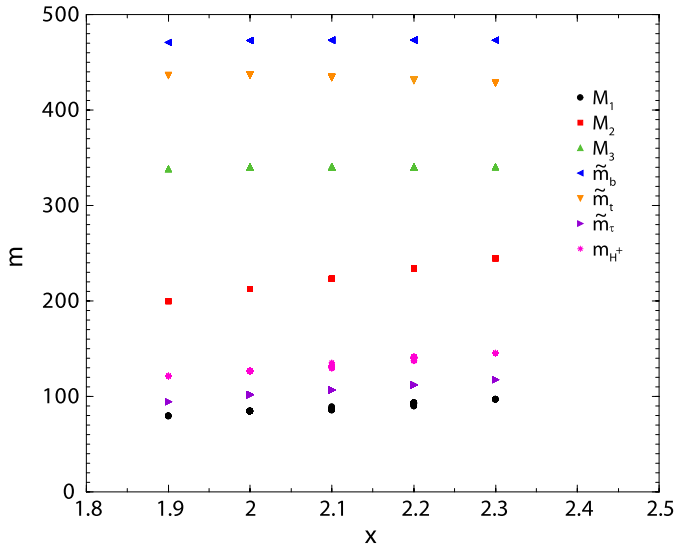
$$\begin{aligned} 50 \text{ TeV} \leq |A_2| \leq 150 \text{ TeV}, \quad 80 \text{ GeV} \leq |A| \leq 500 \text{ GeV}, \\ 0 \leq \phi_3 \leq \pi, \quad -\pi \leq \phi_A \leq 0. \end{aligned} \quad (27)$$

Solutions are found for rather small values of  $|A|$  such as 190–250 GeV, which satisfy  $|A| > |\mu|$ . The desired relation  $|M_1| < |\mu| < |M_2|$  is also satisfied.

In Fig. 1 we show the allowed regions in the  $(\phi_3, \phi_A)$  plane for various values of  $x$  ( $\equiv |A_2|/|A_1|$ ), which satisfy all the EDM constraints of the electron, the neutron and the mercury atom. The imposed constraints restrict the regions of  $x$  to  $1.9 \lesssim x \lesssim 2.3$ . Since the values of  $\phi_3$  obtained yield small values for  $\phi_1$  as found from (3), both sectors of the chargino and the neutralino seem to have no large influence of the phases in the gaugino masses. The EDM



**Fig. 1.** Allowed regions in the  $(\phi_3, \phi_A)$  plane that satisfy the imposed conditions including the EDM constraints



**Fig. 2.** Masses in the mass spectrum of superpartners at the weak scale as functions of  $x$

constraint of the electron is considered to be satisfied without its help. As long as the charginos are heavier than the neutralinos, cancellation between them can occur. In fact, this is satisfied in the present solutions. On the other hand, the phase  $\phi_3$  of the gluino mass affects the EDMs of the neutron and the mercury atom through the gluino contribution. It happens to cause the cancellation for the EDM of the neutron and the mercury atom. In fact, the values of  $\phi_3 - \phi_A$  obtained here may bring about the suppression for the gluino contribution as found in (25). This seems to suggest that the  $CP$  phases in the gaugino sector play a crucial role in satisfying the EDM constraints even in the case of large  $\phi_A$  and  $\phi_\mu$ .

In order to show the features of the SUSY breaking for these solutions, we show the mass spectrum of some superpartners as a function of  $x$  in Fig. 2. They are deter-

<sup>5</sup> We use  $m_t = 174.3 \text{ GeV}$  in this analysis.

mined through the values of  $|A_1|$  and  $|A_2|$  as found in (2) and (4). For the sfermion masses  $\tilde{m}_t$ ,  $\tilde{m}_b$  and  $\tilde{m}_\tau$ , we plot smaller mass eigenvalues. The mass ratio of the chargino to the neutralino can be much larger than that in the ordinary GMSB case ( $x = 1$ ). It is also remarkable that the gluino can be lighter than the squarks. The neutralino is the lightest superpartner except for the gravitino. The mass of the charged Higgs scalar takes a value in the range of 120–150 GeV.

### 3.3 Phenomenology in the Higgs sector

The allowed parameter regions obtained from the EDM constraints generally require small values for  $\tan\beta$ . However, as is well known in the  $CP$  conserving case, the small  $\tan\beta$  predicts the small value of the lightest neutral Higgs mass in the MSSM. Then it can be a serious obstacle to the present solutions for the EDM constraints. It is an important issue to check whether our model can be consistent with the constraints from the present Higgs search [48]. In various works [19–25], it has been shown that the  $CP$  phases in the SUSY breaking parameters could largely change both the Higgs mass eigenvalues and their couplings to the gauge bosons and the fermions. It happens due to the mixings among the  $CP$ -even and  $CP$ -odd Higgs scalars. In the recent analysis of the  $CP$  violating benchmark model CPX with a certain top quark mass, the combined LEP data seem to give no universal lower bound for the lightest neutral Higgs mass, although they can restrict the  $\tan\beta$  to be larger than 2.6 [48]. In the present model, a similar feature may also be found for the parameter region derived from the EDM constraints, and it can be consistent with the present experimental data for the Higgs sector.

In order to study this aspect, we follow the one-loop effective potential method discussed in [21], in which the one-loop effective potential is expanded by the operators up to the fourth order and the effective Higgs quartic couplings are analytically determined. Our EDM study suggests that the small  $\tan\beta$  is favorable and also both  $|A|$  and  $|\mu|$  tend to be smaller than the soft scalar masses of the left- and right-handed stops. These features seem to make the usage of this method valid for the present analysis. In our model the gaugino masses are non-universal and then there can be physical  $CP$  phases in the gaugino masses in addition to those in the  $\mu$  and  $A$  parameters. This is a situation different from that in [21]. The gaugino phases could contribute to the one-loop effective potential mainly through the neutralino and chargino loops. However, since these  $CP$  violating corrections to the effective potential are considered to be smaller than the one coming from the stop contribution, we neglect them in this study, and we directly apply the formulas in [21] to this analysis.

In the following part, we focus our study on the mass eigenvalues of the Higgs scalars and the couplings between the Higgs scalars and the gauge bosons. They can be represented by using the Higgs quartic effective couplings  $\lambda_{1-7}$ . These definitions and their analytical formulas [21] are presented in the appendix. If we impose the potential minimum conditions, we can write the neutral Higgs mass ma-

trix in the form as

$$\mathcal{M}_0^2 = \begin{pmatrix} \mathcal{M}_S^2 & (\mathcal{M}_{PS}^2)^T \\ \mathcal{M}_{PS}^2 & M_a^2 \end{pmatrix}, \quad (28)$$

where  $\mathcal{M}_S^2$  is a  $2 \times 2$  mass matrix for the  $CP$ -even Higgs scalars and  $\mathcal{M}_{PS}^2$  is a  $1 \times 2$  matrix representing mixing among the  $CP$ -odd and  $CP$ -even Higgs scalars. These submatrices can be expressed as

$$\begin{aligned} \mathcal{M}_S^2 &= M_a^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \\ &+ 2v^2 \begin{pmatrix} -2 \left( \lambda_1 c_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) s_\beta^2 + \text{Re}(\lambda_6 e^{i\xi}) s_\beta c_\beta \right) & \\ \lambda_{34} s_\beta c_\beta + \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 & \\ \lambda_{34} s_\beta c_\beta + \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 & \\ -2 \left( \lambda_2 s_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta c_\beta \right) & \end{pmatrix}, \\ \mathcal{M}_{PS}^2 &= 2v^2 \left( \text{Im}(\lambda_5 e^{2i\xi}) s_\beta + \text{Im}(\lambda_6 e^{i\xi}) c_\beta \right. \\ &\quad \left. \text{Im}(\lambda_5 e^{2i\xi}) c_\beta + \text{Im}(\lambda_7 e^{i\xi}) s_\beta \right), \end{aligned} \quad (29)$$

where  $\lambda_{34} = \lambda_3 + \lambda_4$ ,  $s_\beta = \sin\beta$  and  $c_\beta = \cos\beta$ .  $M_a^2$  corresponds to the physical mass of the  $CP$ -odd Higgs scalar in the  $CP$  conserving MSSM, and it can be written as

$$\begin{aligned} M_a^2 &= \frac{1}{s_\beta c_\beta} \left[ \text{Re}(m_{12}^2 e^{i\xi}) + 2v^2 \left\{ 2 \text{Re}(\lambda_5 e^{2i\xi}) s_\beta c_\beta \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \frac{1}{2} \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 \right\} \right]. \end{aligned} \quad (30)$$

The mass of the charged Higgs scalars can be expressed as

$$\begin{aligned} M_{H^\pm}^2 &= \frac{1}{s_\beta c_\beta} \left[ \text{Re}(m_{12}^2 e^{i\xi}) \right. \\ &\quad \left. + 2v^2 \left\{ \frac{1}{2} \lambda_4 s_\beta c_\beta + \text{Re}(\lambda_5 e^{2i\xi}) s_\beta c_\beta \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \frac{1}{2} \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 \right\} \right]. \end{aligned} \quad (31)$$

The Higgs couplings to the gauge bosons are also changed from those in the  $CP$  conserving case. This occurs due to the mixing among the  $CP$ -even and  $CP$ -odd Higgs scalars, which is induced by  $\mathcal{M}_{PS}^2$ . The interaction Lagrangian for the mass eigenstates of the Higgs scalars  $H_i$  is found to be expressed as

$$\begin{aligned} \mathcal{L}_{HVV} &= g_2 M_W \sum_{i=1}^3 g_{H_i VV} \left( H_i W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} H_i Z_\mu Z^\mu \right), \\ \mathcal{L}_{HHZ} &= \frac{g_2}{2c_W} \sum_{j>i=1}^3 g_{H_i H_j Z} \left( H_i \overset{\leftrightarrow}{\partial}_\mu H_j \right) Z^\mu, \\ \mathcal{L}_{HH^\pm W^\mp} &= \frac{g_2}{2} \sum_{i=1}^3 \left[ g_{H_i H^- W^+} \left( H_i \overset{\leftrightarrow}{\partial}_\mu H^- \right) W^{+\mu} + \text{h.c.} \right]. \end{aligned} \quad (32)$$

In these interaction Lagrangians for the Higgs scalars, each coupling normalized to the value in the standard model can be written as

$$\begin{aligned} g_{H_i V V} &= c_\beta O_{1i} + s_\beta O_{2i}, \quad (V = W^\pm, Z) \\ g_{H_i H_j Z} &= O_{3i}(c_\beta O_{2j} - s_\beta O_{1j}) - O_{3j}(c_\beta O_{2i} - s_\beta O_{1i}), \\ g_{H_i H^- W^+} &= c_\beta O_{2i} - s_\beta O_{1i} + iO_{3i}, \end{aligned} \quad (33)$$

where  $O_{ij}$  is the element of the orthogonal matrix that relates the mass eigenstates  $H_i$  to the weak eigenstates. It is defined as the diagonalization matrix for  $\mathcal{M}_0^2$  in such a way that

$$O^T \mathcal{M}_0^2 O = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2), \quad (34)$$

where the mass eigenvalues  $m_{H_i}^2$  for the eigenstates  $H_i$  satisfy the relation such as  $m_{H_1}^2 \leq m_{H_2}^2 \leq m_{H_3}^2$ . Since there are the following relations among these neutral Higgs couplings:

$$g_{H_k V V} = \varepsilon_{ijk} g_{H_i H_j Z}, \quad \sum_{i=1}^3 g_{H_i V V}^2 = 1, \quad (35)$$

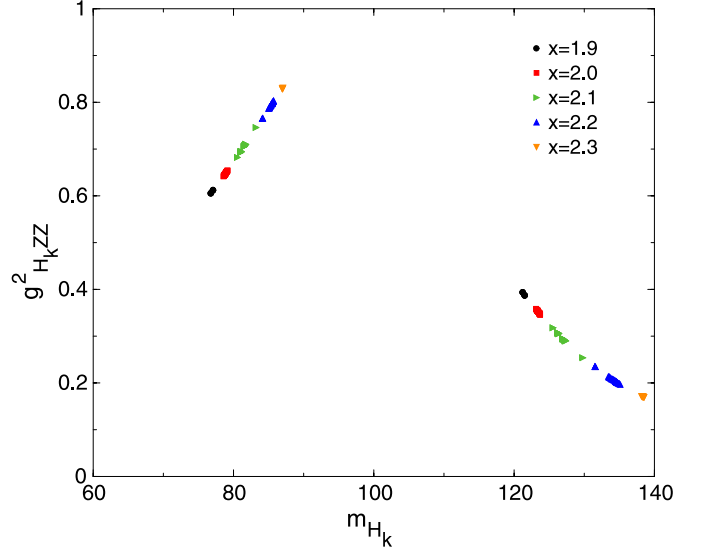
all of the couplings of the neutral Higgs scalars to the gauge bosons can be completely determined by the two values of  $g_{H_i Z Z}$ , for example [56, 57].

As mentioned already, the  $CP$  violating effect in the Higgs sector appears through the mixing  $\mathcal{M}_{PS}^2$  between the  $CP$ -even and  $CP$ -odd Higgs scalars. If we use the analytic formulas for the quartic couplings  $\lambda_{5,6,7}$  in the appendix, we find that the order of these off-diagonal elements are estimated as

$$\begin{aligned} \mathcal{M}_{SP}^2 &\simeq O\left(\frac{m_t^4 \text{Im}(A\mu)}{v^2 64\pi^2 M_S^2}\right) \\ &= O\left(\frac{v^2 |A||\mu| \sin \phi_{CP}}{64\pi^2 M_S^2} \left(\frac{\tan^2 \beta}{1 + \tan^2 \beta}\right)^2\right), \end{aligned} \quad (36)$$

where  $\phi_{CP} = \phi_A + \phi_\mu$ , which is a measure for  $CP$  violation in the Higgs sector. If  $\text{Im}(A\mu)$  can have large values, they may be so large as to have crucial effects on the composition of the mass eigenstates of the neutral Higgs scalars. Thus, the larger values of  $|\mu|$ ,  $|A|$  and  $\phi_{CP}$  constitute interesting parameter regions, in which the  $CP$  violating effects on the Higgs sector are substantial. On the other hand, as discussed in [21], the  $CP$  violating effects on the Higgs sector also tend to be enhanced in the case that the charged Higgs mass  $M_{H^\pm}$  takes a small value<sup>6</sup>. In the present model,  $A$ ,  $B$  and  $\mu$  are free parameters. Since they are not directly related to other SUSY breaking parameters, such as the gaugino masses and the sfermion masses, we can study their interesting regions without making large changes in the mass spectrum of the gauginos and the

<sup>6</sup> This is expected to be realized for the case of a small value of  $m_{12}^2 (= B\mu)$ . However, if the top quark mass is larger, the  $CP$  violating effect seems to appear independently of the charged Higgs mass [48].



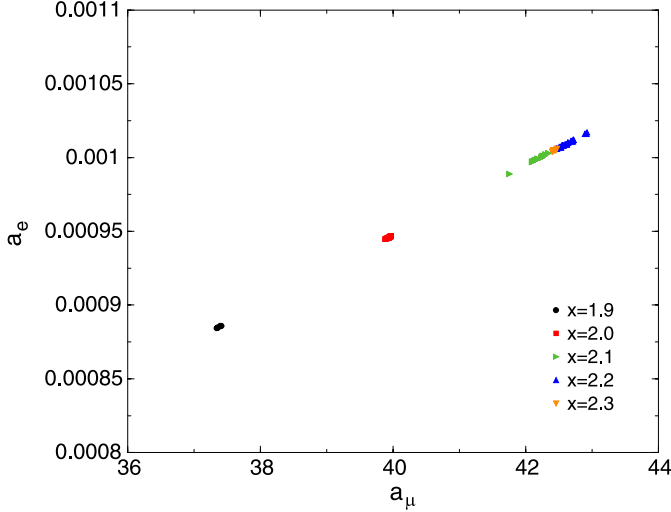
**Fig. 3.** The mass eigenvalues and the coupling constants with the gauge bosons of the neutral Higgs scalars  $H_1$  and  $H_3$

sfermions as long as  $\tilde{m}_f > |A|$ ,  $|\mu|$  is satisfied. However, we should note that the EDM constraints tend to favor small values of  $\tan \beta$ ,  $|A|$  and  $|\mu|$  as partially seen in (25), for example. Thus, the EDM constraints may make the  $CP$  violating effects in the Higgs sector small even in the case with large  $\phi_{CP}$ . Although the cases where  $|A|$  and  $|\mu|$  are not large but  $\phi_{CP}$  is  $O(1)$  seem to be promising in the present context, the situation is subtle and a detailed numerical study is required to clarify this point.

We calculate both the Higgs mass eigenvalues and the Higgs couplings for the parameter sets obtained in the previous part. In Fig. 3 we plot the mass eigenvalues of the neutral Higgs scalars  $H_k$  and their coupling constants  $g_{H_k Z Z}^2$  with the gauge bosons. Both the lightest neutral Higgs scalar  $H_1$  and the heaviest one  $H_3$  are plotted in the same figure for each value of  $x$ . Since (35) is satisfied among the couplings,  $g_{H_2 Z Z}^2$  is negligible in the present case. The mass eigenvalues  $m_{H_k}$  are increasing functions of  $x$ . If we combine this figure with Fig. 1, we can see that they are affected largely by the phase  $\phi_{CP}$ . Since these Higgs mass eigenvalues take rather small values, the model might be considered to already have been excluded by the Higgs search at LEP. However, the Higgs couplings are also influenced largely by this  $\phi_{CP}$ , as observed in Fig. 3. Figure 3 shows that the  $H_1$  coupling  $g_{H_1 Z Z}^2$  can be much smaller than the MSSM one. The values of  $m_{H_1}$  and  $g_{H_1 Z Z}^2$  shown in Fig. 3 seem to be marginal against the LEP2 data [48].

We could only say that our solutions for the EDM constraints might be consistent with the present Higgs phenomenology on the basis of our analysis. However, our results suggest that the validity of the model can be checked if new experiments start at LHC, anyway. Our analysis can also give several predictions for the relevant physical quantities. As a good example, we estimate the SUSY contributions  $a_\mu$  and  $a_e$  to the anomalous magnetic moment of the muon and the electron. The results are shown in Fig. 4. Both  $a_\mu$  and  $a_e$  are plotted in units of  $10^{-11}$ . The predicted





**Fig. 4.** The SUSY contributions to  $g-2$  of the muon and the electron expected for the obtained solutions

values of  $g-2$  of the muon seem to be in the interesting regions for the present experimental data.

## 4 Summary

Non-universality of the gaugino masses may potentially cause interesting phenomenology in various respects at the weak scale. We have considered the extended gauge mediation SUSY breaking scenario as a concrete example that could realize non-universal gaugino masses. In this model the  $CP$  phases can remain in the gaugino sector as the physical phases after  $R$ -transformation. In addition to this aspect, the model has several features different from the usual MSSM or the ordinary gauge mediation SUSY breaking. For example, the  $SU(2)_L$  non-singlet superpartners tend to be heavier than the  $SU(2)_L$  singlet ones, whether they are colored or not. The right-handed stop becomes rather light and the neutralino can be lighter than the stau. These features can affect phenomenology in various ways to give results different from the ordinary MSSM.

We have calculated the effect of the  $CP$  phases on the EDM of the mercury atom, the electron and the neutron by solving the RGEs for the soft SUSY breaking parameters. As a result of this analysis, we have found that the experimental bounds for these EDMs could be simultaneously satisfied without assuming heavy superpartners with a mass of  $O(1)$  TeV even in the case that the soft SUSY breaking parameters have large  $CP$  phases. The effective cancellation among the contributions from the gluino, the neutralino, and the chargino makes it possible for them to satisfy the experimental constraints. In this cancellation, the  $CP$  phases in the gaugino sector seem to play a crucial role. Although this kind of phenomena has already been suggested in several works, we have shown this in the concrete model with the definite spectrum of the superpartners.

The Higgs sector could also be affected by the existence of large  $CP$  phases in the soft SUSY breaking parameters. Since the  $CP$ -even Higgs scalars mix with the  $CP$ -odd Higgs scalar, the lightest neutral Higgs mass and its couplings to the gauge bosons could largely be modified from those in the  $CP$  invariant case. We have studied these aspects in the parameter regions where the EDM constraints are satisfied. From this study, we have found that our model might be consistent with the present data obtained from the Higgs search at LEP2. The validity of the model will be checked at LHC.

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## Appendix

The effective Lagrangian that describes the most general  $CP$ -violating Higgs potential of the MSSM is given by

$$\begin{aligned} \mathcal{L} = & \mu_1^2 (\Phi_1^\dagger \Phi_1) + \mu_2^2 (\Phi_2^\dagger \Phi_2) + m_{12}^2 (\Phi_1^\dagger \Phi_2) + m_{12}^{*2} (\Phi_2^\dagger \Phi_1) \\ & + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_5^* (\Phi_2^\dagger \Phi_1)^2 \\ & + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_6^* (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_1) \\ & + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \lambda_7^* (\Phi_2^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1), \quad (\text{A.1}) \end{aligned}$$

where  $\Phi_{1,2}$  are related to the scalar components  $H_{1,2}$  of the Higgs superfields  $\hat{H}_{1,2}$  through  $H_1 = i\tau_2 \Phi_1^*$  and  $H_2 = \Phi_2$ . At the tree level, the coefficients in (A.1) are represented as

$$\begin{aligned} \mu_1^2 = & -m_1^2 - |\mu|^2, \quad \mu_2^2 = -m_2^2 - |\mu|^2, \quad m_{12}^2 = B\mu, \\ \lambda_1 = & \lambda_2 = -\frac{1}{8} (g_2^2 + g_1^2), \quad \lambda_3 = -\frac{1}{4} (g_2^2 - g_1^2), \quad \lambda_4 = \frac{1}{2} g_2^2, \\ \lambda_5 = & \lambda_6 = \lambda_7 = 0. \quad (\text{A.2}) \end{aligned}$$

Taking account of radiative corrections due to the trilinear Yukawa couplings between the Higgs scalars and stops/sbottoms, the quartic couplings  $\lambda_{5,6,7}$  generally have complex nonzero values. If we assume that  $M_S$  is a SUSY breaking scale, analytic expressions of these quartic couplings are given by [21]

$$\begin{aligned} \lambda_1 = & -\frac{g_2^2 + g_1^2}{8} \left( 1 - \frac{3}{8\pi^2} h_b^2 t \right) \\ & - \frac{3}{16\pi^2} h_b^4 \left[ t + \frac{1}{2} X_b + \frac{1}{16\pi^2} \left( \frac{3}{2} h_b^2 + \frac{1}{2} h_t^2 - 8g_3^2 \right) (X_b t + t^2) \right] \\ & + \frac{3}{192\pi^2} h_t^2 \frac{|\mu|^4}{M_S^4} \left[ 1 + \frac{1}{16\pi^2} (9h_t^2 - 5h_b^2 - 16g_3^2) t \right], \end{aligned}$$

$$\begin{aligned}
\lambda_2 &= -\frac{g_2^2 + g_1^2}{8} \left(1 - \frac{3}{8\pi^2} h_t^2 t\right) - \frac{3}{16\pi^2} h_t^4 \\
&\times \left[ t + \frac{1}{2} X_t + \frac{1}{16\pi^2} \left(\frac{3}{2} h_t^2 + \frac{1}{2} h_b^2 - 8g_3^2\right) (X_t t + t^2) \right] \\
&+ \frac{3}{192\pi^2} h_b^2 \frac{|\mu|^4}{M_S^4} \left[1 + \frac{1}{16\pi^2} (9h_b^2 - 5h_t^2 - 16g_3^2) t\right], \\
\lambda_3 &= -\frac{g_2^2 - g_1^2}{8} \left[1 - \frac{3}{16\pi^2} (h_t^2 + h_b^2) t\right] - \frac{3}{8\pi^2} h_t^2 h_b^2 \\
&\times \left[ t + \frac{1}{2} X_{tb} + \frac{1}{16\pi^2} (h_t^2 + h_b^2 - 8g_3^2) (X_{tb} t + t^2) \right] \\
&- \frac{3}{96\pi^2} h_t^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_t|^2}{M_S^4}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t\right] \\
&- \frac{3}{96\pi^2} h_b^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_b|^2}{M_S^4}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t\right], \\
\lambda_4 &= \frac{g_2^2}{2} \left[1 - \frac{3}{16\pi^2} (h_t^2 + h_b^2) t\right] + \frac{3}{8\pi^2} h_t^2 h_b^2 \\
&\times \left[ t + \frac{1}{2} X_{tb} + \frac{1}{16\pi^2} (h_t^2 + h_b^2 - 8g_3^2) (X_{tb} t + t^2) \right] \\
&- \frac{3}{96\pi^2} h_t^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_t|^2}{M_S^4}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t\right] \\
&- \frac{3}{96\pi^2} h_b^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_b|^2}{M_S^4}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t\right], \\
\lambda_5 &= \frac{3}{192\pi^2} h_t^4 \frac{\mu^2 A_t^2}{M_S^4} \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t\right] \\
&+ \frac{3}{192\pi^2} h_b^4 \frac{\mu^2 A_b^2}{M_S^4} \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t\right], \\
\lambda_6 &= -\frac{3}{96\pi^2} h_t^4 \frac{|\mu|^2 \mu A_t}{M_S^4} \left[1 + \frac{1}{16\pi^2} \left(\frac{15}{2} h_t^2 - \frac{7}{2} h_b^2 - 16g_3^2\right) t\right] \\
&+ \frac{3}{96\pi^2} h_b^4 \frac{\mu}{M_S} \left(\frac{6A_b}{M_S} - \frac{|A_b|^2 A_b}{M_S^3}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} \left(\frac{9}{2} h_b^2 - \frac{1}{2} h_t^2 - 16g_3^2\right) t\right], \\
\lambda_7 &= -\frac{3}{96\pi^2} h_b^4 \frac{|\mu|^2 \mu A_b}{M_S^4} \left[1 + \frac{1}{16\pi^2} \left(\frac{15}{2} h_b^2 - \frac{7}{2} h_t^2 - 16g_3^2\right) t\right] \\
&+ \frac{3}{96\pi^2} h_t^4 \frac{\mu}{M_S} \left(\frac{6A_t}{M_S} - \frac{|A_t|^2 A_t}{M_S^3}\right) \\
&\times \left[1 + \frac{1}{16\pi^2} \left(\frac{9}{2} h_t^2 - \frac{1}{2} h_b^2 - 16g_3^2\right) t\right]. \tag{A.3}
\end{aligned}$$

In these formulas, the following definitions are used:

$$t = \ln\left(\frac{M_S^2}{\bar{m}_t^2}\right), \quad h_t = \frac{m_t(\bar{m}_t)}{v \sin \beta}, \quad h_b = \frac{m_b(\bar{m}_t)}{v \cos \beta},$$

$$\begin{aligned}
X_t &= \frac{2|A_t|^2}{M_S^2} \left(1 - \frac{|A_t|^2}{12M_S^2}\right), \quad X_b = \frac{2|A_b|^2}{M_S^2} \left(1 - \frac{|A_b|^2}{12M_S^2}\right), \\
X_{tb} &= \frac{|A_t|^2 + |A_b|^2 + 2\operatorname{Re}(A_b^* A_t)}{2M_S^2} - \frac{|\mu|^2}{M_S^2} - \frac{||\mu|^2 - A_b^* A_t|^2}{6M_S^4}, \tag{A.4}
\end{aligned}$$

where  $\bar{m}_t$  is the pole mass of the top quark, which can be related to the running mass  $m_t$  by

$$m_t(\bar{m}_t) = \frac{\bar{m}_t}{1 + \frac{4}{3\pi} \alpha_3(\bar{m}_t)}. \tag{A.5}$$

We assume that the SUSY breaking scale  $M_S^2$  is defined as the arithmetic average of the squared stop mass eigenvalues in the numerical calculation of the Higgs sector.

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